

Chapter 7

Transformations

Section 2

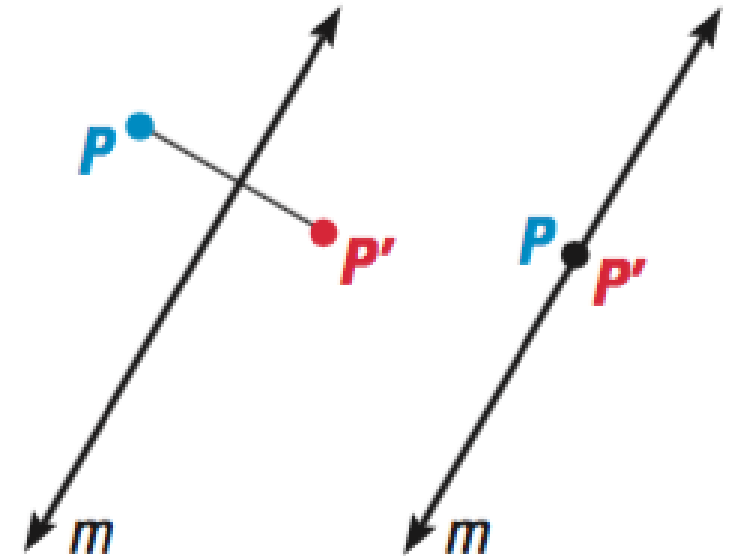
Reflections

GOAL 1: Using Reflections in a Plane

One type of transformation uses a line that acts like a mirror, with an image reflected in the line. This transformation is a **reflection** and the mirror line is the **line of reflection**.

A reflection in a line m is a transformation that maps every point P in the plane to a point P' , so that the following properties are true:

1. If P is not on m , then m is the perpendicular bisector of $\overline{PP'}$.
2. If P is on m , then $P = P'$.



Example 1: Reflections in a Coordinate Plane

Graph the given reflections.

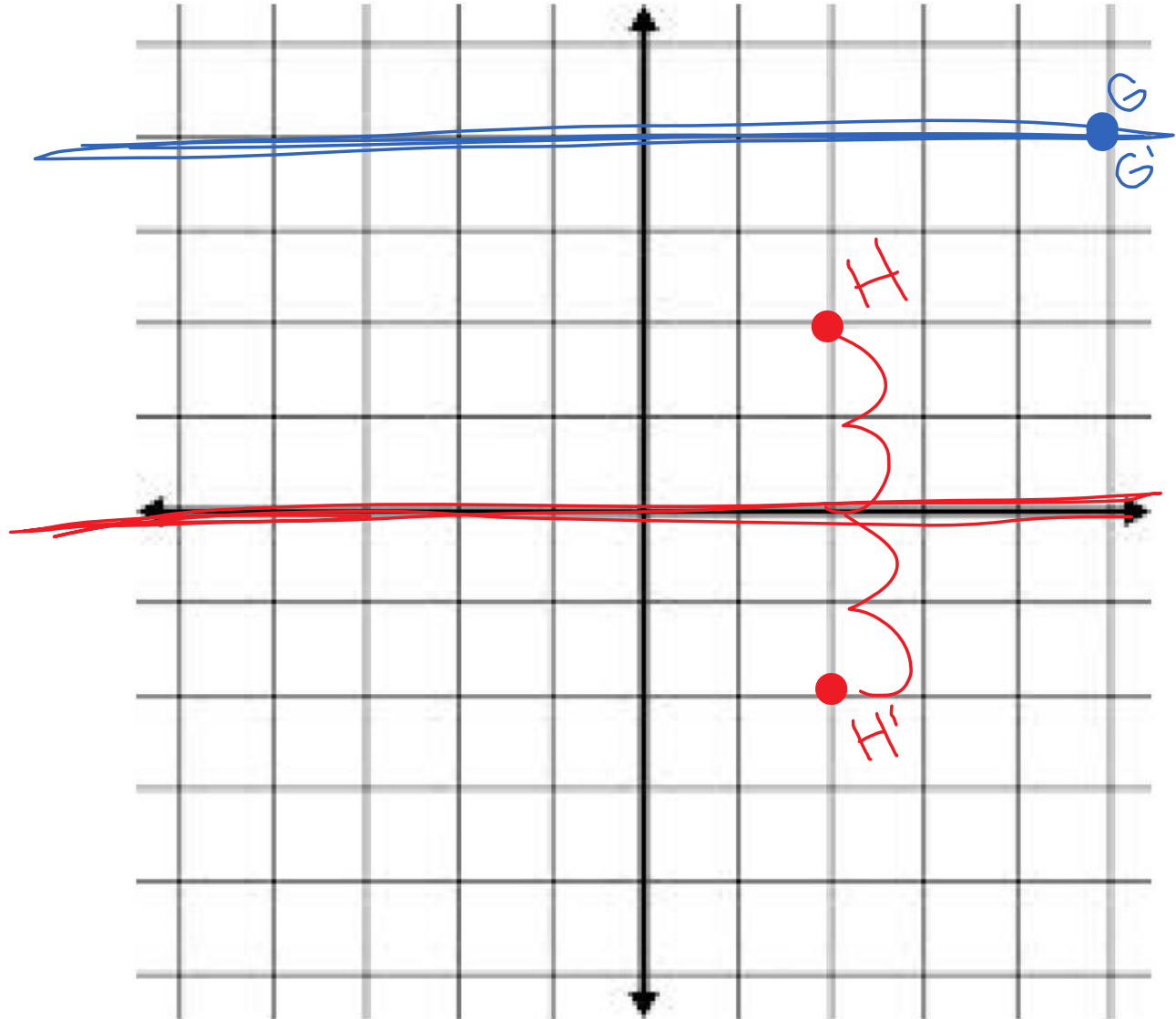
a) $H(2, 2)$ in the x-axis

$$H'(2, -2)$$

b) $G(5, 4)$ in the line $y = 4$.

$$G'(5, 4)$$

(Same point b/c it is
on the line of
reflection)



Reflections in the coordinate axes have the following properties:

1. If (x, y) is reflected in the x -axis, its image is the point $(x, -y)$.
2. If (x, y) is reflected in the y -axis, its image is the point $(-x, y)$.

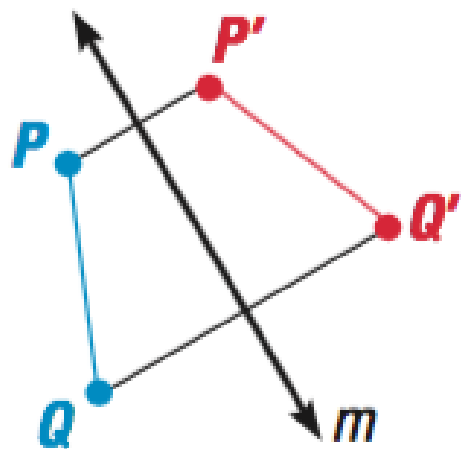
In Lesson 7.1, you learned that an isometry preserves lengths. Theorem 7.1 relates isometries and reflections.

THEOREM

THEOREM 7.1 *Reflection Theorem*

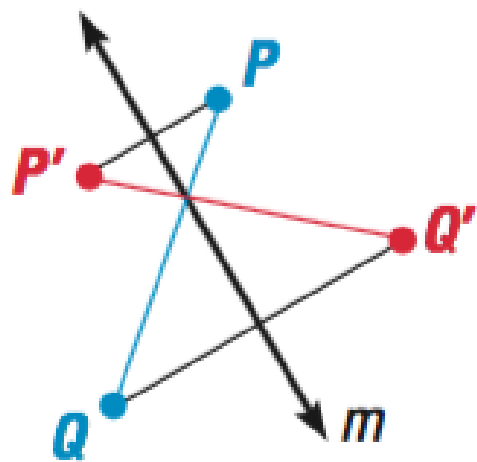
A reflection is an isometry.

To prove the Reflection Theorem, you need to show that a reflection preserves the length of a segment. Consider a segment \overline{PQ} that is reflected in a line m to produce $\overline{P'Q'}$. The four cases to consider are shown below.



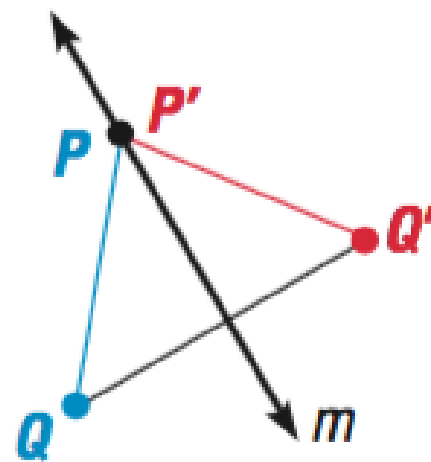
Case 1

P and Q are on the same side of m .



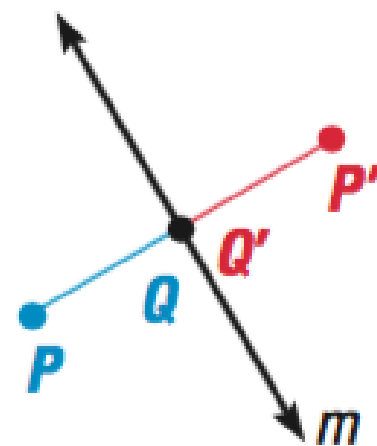
Case 2

P and Q are on opposite sides of m .



Case 3

One point lies on m and \overline{PQ} is not perpendicular to m .



Case 4

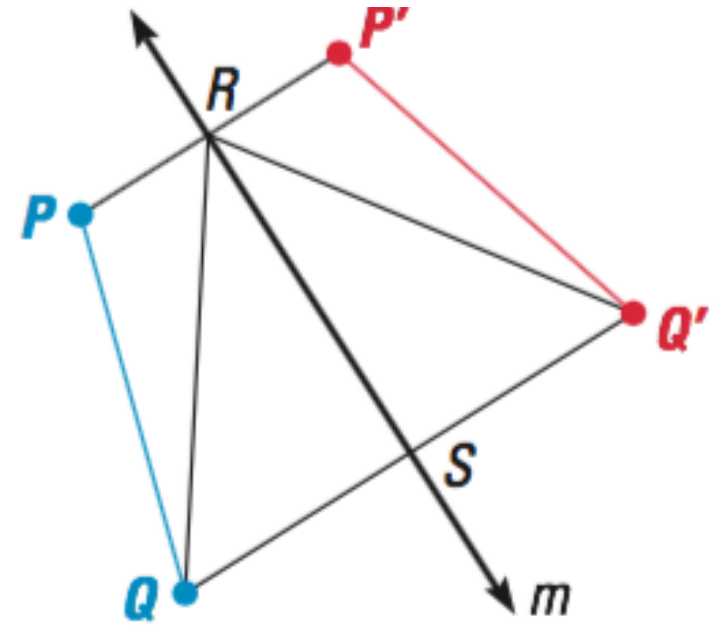
Q lies on m and $\overline{PQ} \perp m$.

Example 2: Proof of Case 1 of Theorem 7.1

GIVEN ► A reflection in m maps P onto P' and Q onto Q' .

PROVE ► $PQ = P'Q'$

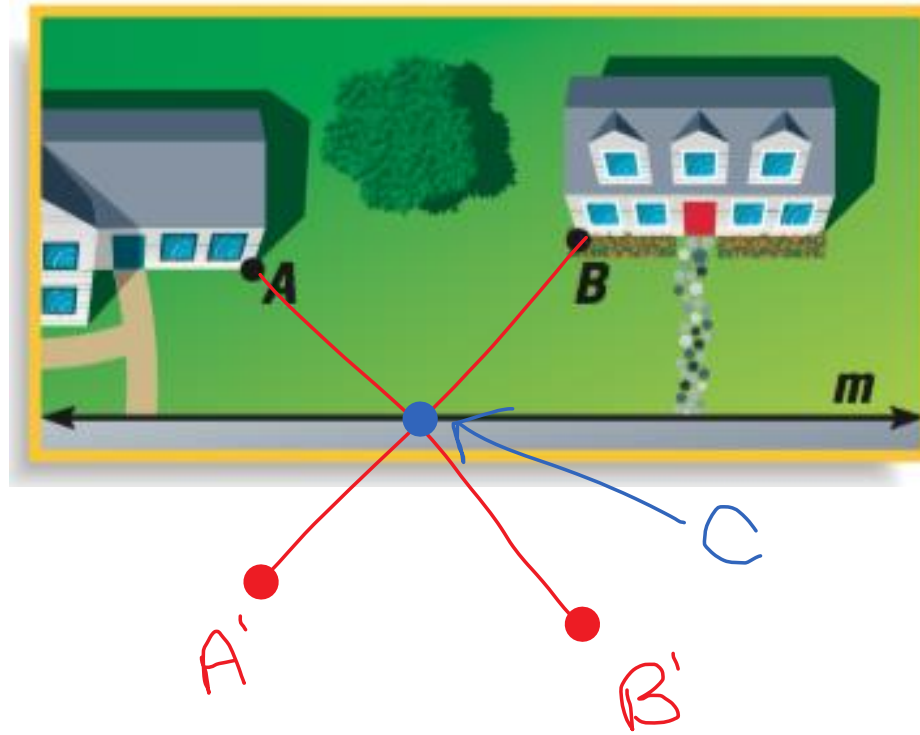
Paragraph Proof For this case, P and Q are on the same side of line m . Draw $\overline{PP'}$ and $\overline{QQ'}$, intersecting line m at R and S . Draw \overline{RQ} and $\overline{RQ'}$.



By the definition of a reflection, $m \perp \overline{QQ'}$ and $\overline{QS} \cong \overline{Q'S}$. It follows that $\triangle RSQ \cong \triangle RSQ'$ using the SAS Congruence Postulate. This implies $\overline{RQ} \cong \overline{RQ'}$ and $\angle QRS \cong \angle Q'RS$. Because \overleftrightarrow{RS} is a perpendicular bisector of $\overline{PP'}$, you have enough information to apply SAS to conclude that $\triangle RQP \cong \triangle RQ'P'$. Because corresponding parts of congruent triangles are congruent, $PQ = P'Q'$.

Example 3: Finding a Minimum Distance

Surveying: Two houses are located on a rural road m , as shown at the right. You want to place a telephone pole on the road at point C so that the length of the telephone cable, $AC + BC$, is a minimum. Where should you locate C ?



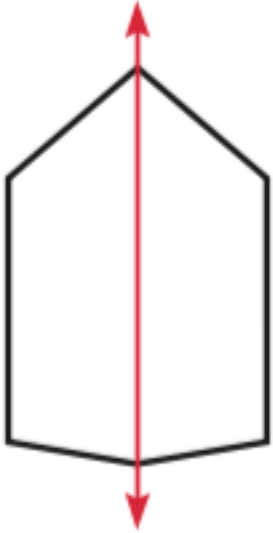
GOAL 2: Reflections and Line Symmetry

A figure in the plane has a **line of symmetry** if the figure can be mapped onto itself by a reflection in the line.

Example 4: Finding Lines of Symmetry

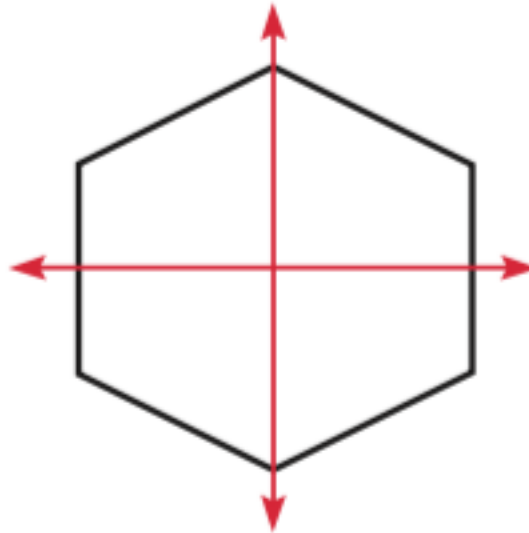
Hexagons can have different lines of symmetry depending on their shape.

a.



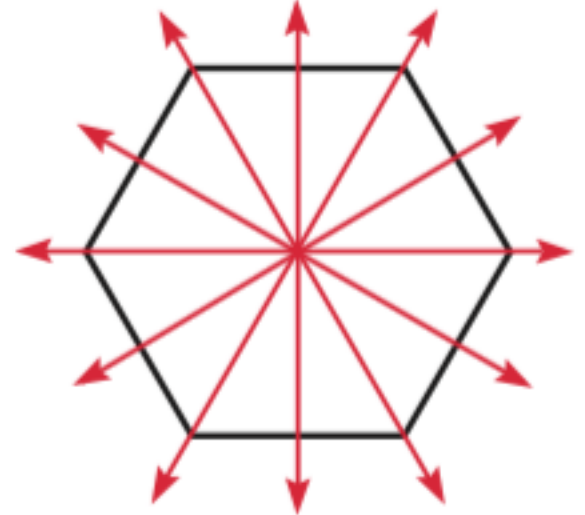
This hexagon has only one line of symmetry.

b.



This hexagon has two lines of symmetry.

c.



This hexagon has six lines of symmetry.

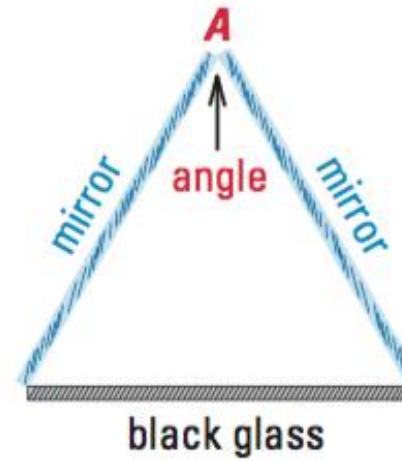
When it is REGULAR, the number of sides = the number of lines of symmetry

Example 5: Identifying Reflections

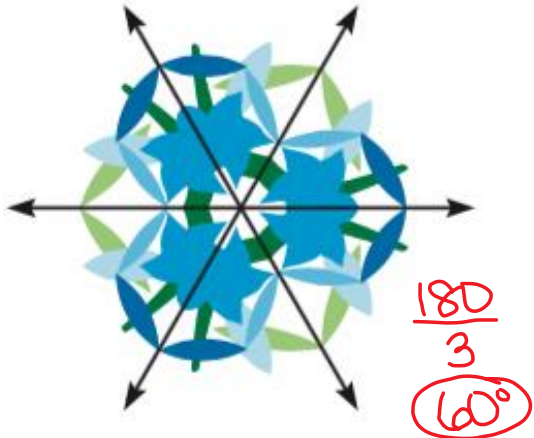
KALEIDOSCOPES Inside a kaleidoscope, two mirrors are placed next to each other to form a V, as shown at the right. The angle between the mirrors determines the number of lines of symmetry in the image. The formula below can be used to calculate the angle between the mirrors, A , or the number of lines of symmetry in the image, n .

$$n(m\angle A) = 180^\circ \Rightarrow \angle A = \frac{180}{n}$$

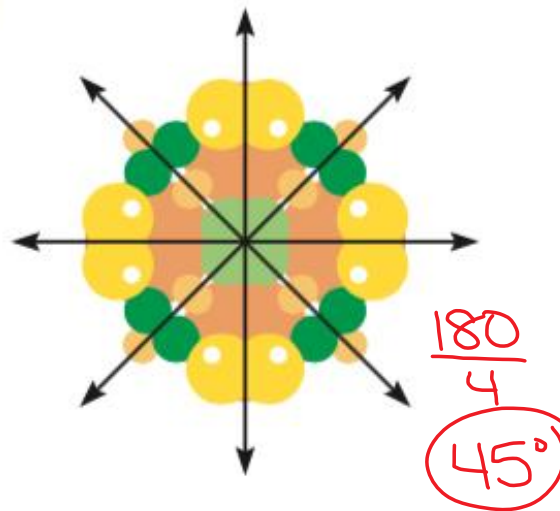
Use the formula to find the angle that the mirrors must be placed for the image of a kaleidoscope to resemble the design.



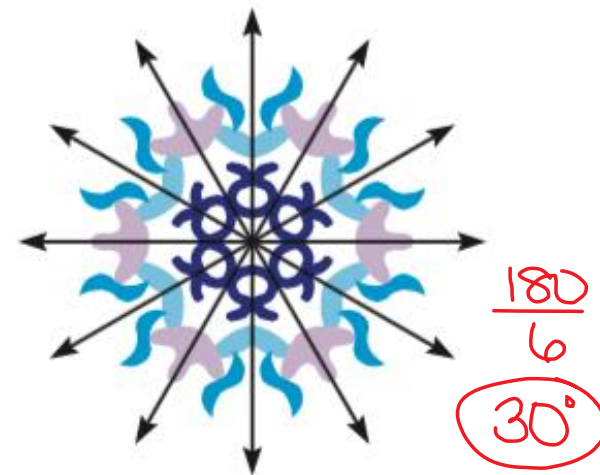
a.



b.



c.



EXIT SLIP